A symbolic approach to perturbed second degree forms: study of the case of perturbed Chebyshev polynomials

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Abstract

By means of a general method and the corresponding symbolic algorithm *PSDF* [2, 3] based on Stieltjes equations, we are able to explicit several semi–classical properties of perturbed second degree forms namely: the Stieltjes function, the Stieltjes equation, the functional equation, the class, a structure relation and the second order linear differential equation. Applying the algorithm *PSDF* to the Chebyshev form of second kind, we achieve to explicit the above mentioned properties and the generating functions for perturbations of several fixed orders [2, 3] generalizing existent results in literature. From these properties, we can easily derive similar ones for the other three forms of Chebyshev [2].

Also, we consider the problem of finding the connection coefficients [4, 5] that allow to write the perturbed of the Chebyshev polynomials of second kind in terms of the original sequence, and in terms of the canonical basis. Starting from some symbolic results produced by the software *CCOP* [6, 7], we achieve to generalize known formulas [1] for any order of perturbation.

References

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